Contents

Preface xiii
Foreword xvii

PART 1 Mathematical Aspects 1

1 Forms of Wiener-Hopf equations 3

1.1 The basic Wiener-Hopf equation 3
1.1.1 An electromagnetic example: The half-plane problem 5
1.2 Modified W-H equations (MWHE) 7
1.2.1 Longitudinally modified W-H equations 7
1.2.2 Transversely modified W-H equations 9
1.2.3 The incomplete Wiener-Hopf equations 10
1.3 Generalized W-H equations 12
1.3.1 An electromagnetic example: The PEC wedge problem 12
1.3.2 An electromagnetic example: The dielectric wedge problem 13
1.4 The Hilbert-Riemann problem 14
1.5 Reduction of W-H equations to the classical form 14
1.5.1 Reduction of the transversely modified W-H equations to CWHE 14
1.5.2 Reduction of the longitudinally modified W-H equations to CWHE 15
1.5.3 The Hilbert-Riemann equations 16
1.5.4 Generalized Wiener-Hopf equations 16
1.6 From Wiener-Hopf equations to Fredholm integral equations in the spectral domain 17
1.7 Fundamental literature 19

2 The exact solution of Wiener-Hopf equations 21

2.1 Introduction 21

2.2 Additive decomposition 22

2.3 Multiplicative decomposition or factorization 23

2.4 Solution of the W-H equation 24

2.4.1 Solution of the nonhomogeneous equation 24

2.4.2 Remote source 27

2.5 Unbounded plus and minus unknowns 29

2.6 Factorized matrices as solutions of the homogeneous Wiener-Hopf problem 29

2.7 Nonstandard factorizations 31

2.8 Extension of the W-H technique to the GWHE 34

2.9 Important mappings for dealing with W-H equations 35

2.9.1 The $\chi = \sqrt{\tau_o^2 - \alpha^2}$ mapping 35

2.9.2 The $\alpha = -\tau_o \cos w$ mapping 36

3 Functions decomposition and factorization 45

3.1 Decomposition 45

3.1.1 Example 1 47

3.1.2 Decomposition of an even function 51

3.1.3 Numerical decomposition 51

3.1.4 Example 1 revisited 53

3.1.5 The case of meromorphic functions 54

3.1.6 Decomposition using rational approximants of the function 55

3.2 Factorization 57

3.2.1 General formula for the scalar case 57

3.2.2 Example 2 57

3.2.3 Example 3 58

3.2.4 Factorization of meromorphic functions 58

3.2.5 Example 4 60

3.2.6 Factorization of kernels involving continuous and discrete spectrum 63

3.3 Decomposition equations in the w – plane 66

3.3.1 Evaluation of the plus functions 66

3.3.2 Evaluation of the minus functions 69

3.3.4 Use of difference equation for function decomposition 73

3.3.5 The W-H equation as difference equation 73

4 Exact matrix factorization 75

4.1 Introduction 75

4.2 Some possibilities to reduce the order of the kernel matrices 76

4.3 Factorization of triangular matrices 78

4.4 Factorization of rational matrices 80

4.4.1 Introduction 80

4.4.2 Matching of the singularities 81

4.4.3 The factorization in the framework of the Fredholm equations 85

4.5 Techniques for solving the factorization problem 86

4.5.1 The logarithmic decomposition 86

4.6 The factorization problem and the functional analysis 92

4.6.1 The iterative method 92

4.6.2 The Fredholm determinant method 93

4.6.3 Factorization of meromorphic matrix kernels with an infinite number of poles 94

4.7 A class of matrices amenable to explicit factorization: matrices having rational eigenvectors 95

4.8 Factorization of a 2×2 matrix 96

4.8.1 The Hurd method 96

4.8.2 The off-diagonal form 98

4.8.3 Reduction of matrices commuting with polynomial matrices to the Daniele matrices 99

4.8.4 Explicit factorization of Daniele matrices 101

4.8.5 The elimination of the offensive behavior for matrices having the Daniele form 104

4.8.6 A relatively simple case 106

4.8.7 The $\sqrt{a(\alpha)/b(\alpha)}$ rational function of α case 108

4.9 The factorization of matrices commuting with rational matrices 110

4.9.1 Introduction 110

4.9.2 Matrix of order two commuting with polynomial matrices 111

4.9.3 Explicit expression of $\psi_i(\alpha)$ in the general case 113

4.9.4 Asymptotic behavior of the logarithmic representation of $-l(\alpha)P^{-1}(\alpha) + 1$ 117

4.9.5 Asymptotic behavior of the decomposed $\psi_{i\pm}(\alpha)$ 118

4.9.6 A procedure to eliminate the exponential behavior 120

4.9.7 On the reduction of the order of the system 124

4.9.8 The nonlinear equations as a Jacobi inversion problem 125

4.9.9 Weakly factorization of a matrix commuting with a polynomial matrix 127

5 Approximate solution: The Fredholm factorization 129

5.1 The integral equations in the α – plane 129

5.1.1 Introduction 129

5.1.2 Source pole α_o with positive imaginary part 130

5.1.3 Analytical validation of a particular W-H equation 131

5.1.4 A property of the integral in the Fredholm equation 132

5.1.5 Numerical solution of the Fredholm equations 134

5.1.6 Analytic continuation outside the integration line 141

5.2 The integral equations in the w – plane 143

5.3 Additional considerations on the Fredholm equations 146

5.3.1 Presence of poles of the kernel in the warped region 146

5.3.2 The Fredholm factorization for particular matrices 147

5.3.3 The Fredholm equation relevant to a modified kernel 147

6 Approximate solutions: Some particular techniques 149

6.1 The Jones method for solving modified W-H equations 149

6.1.1 Introduction 149

6.1.2 Longitudinal modified W-H equation 149

6.1.3 Transversal modified W-H equation 152

6.2 The Fredholm factorization for particular matrices 153

6.3 Rational approximation of the kernel 161

6.3.1 Pade approximants 161

6.3.2 An interpolation approximant method 163

6.4 Moment method 167

6.4.1 Introduction 167

6.4.2 Stationary properties of the solutions with the moment method 169

6.4.3 An electromagnetic example: the impedance of a wire antenna in free space 173

6.5 Comments on the approximate methods for solving W-H equations 175

PART 2 Applications 177

7 The half-plane problem 179

7.1 Wiener-Hopf solution of discontinuity problems in plane-stratified regions 179

7.2 Spectral transmission line in homogeneous isotropic regions 180

7.2.1 Circuital considerations 181

7.2.2 Jump of voltage or current in a section where it is present a discontinuity 182
7.2.3 Jump of voltage or current in a section where a concentrated source is present 182
7.3 Wiener-Hopf equations in the Laplace domain 183
7.4 The PEC half-plane problem 185
7.4.1 E-polarization case 185
7.4.2 Far-field contribution 188
7.5 Skew incidence 191
7.6 Diffraction by an impedance half plane 197
7.6.1 Deduction of W-H equations in diffraction problems by impenetrable half-planes 197
7.6.2 Presence of isotropic impedances Z_a and Z_b 200
7.7 The general problem of factorization 203
7.7.1 The case of symmetric half-plane 205
7.7.2 The case of opposite diagonal impedances $\mathbf{Z}_b = -\mathbf{Z}_a$ 206
7.8 The jump or penetrable half-plane problem 206
7.9 Full-plane junction at skew incidence 207
7.10 Diffraction by an half plane immersed in arbitrary linear medium 208
7.10.1 Transverse equation in an indefinite medium 208
7.10.2 Field equations in the Fourier domain 210
7.10.3 The W-H equation for a PEC or a PMC half-plane immersed in a homogeneous linear arbitrary medium 216
7.11 The half-plane immersed in an arbitrary planar stratified medium 220

8 Planar discontinuities in stratified media 223

8.1 The planar waveguide problem 223
8.1.1 The E-polarization case 223
8.1.2 Source constituted by plane wave 225
8.1.3 Source constituted by an incident mode 227
8.1.4 The skew plane wave case 228
8.2 The reversed half-planes problem 230
8.2.1 The E-polarization case 230
8.2.2 Qualitative characteristics of the solution 231
8.2.3 Numerical evaluation of the electromagnetic field 232
8.2.4 Numerical solution of the W-H equations 233
8.2.5 Source constituted by a skew plane wave 237

8.3 The three half-planes problem 244
8.3.1 The E-polarization case (normal incidence case) 244
8.3.2 The skew incidence case 247
8.4 Arrays of parallel wire antennas in stratified media 248
8.4.1 The single antenna case 248
8.4.2 The W-H equations of an array of wire antennas 250
8.4.3 Spectral theory of transmission lines constituted by bundles of wires 254
8.5 Spectral theory of microstrip and coplanar transmission lines 254
8.5.1 Coplanar line with two strips 254
8.5.2 The shielded microstrip transmission line 260
8.6 General W-H formulation of planar discontinuity problems in arbitrary stratified media 261
8.6.1 Formal solution with the factorization method 263
8.6.2 The method of stationary phase for multiple integrals 267
8.6.3 The circular aperture 268
8.6.4 The quarter plane problem 272

9 Wiener-Hopf analysis of waveguide discontinuities 279

9.1 Marcuvitz-Schwinger formalism 279
9.1.1 Example 1 280
9.1.2 Example 2 283
9.2 Bifurcation in a rectangular waveguide 285
9.3 The junction of two waveguides 287
9.4 A general discontinuity problem in a rectangular waveguide 289
9.5 Radiation from truncated circular waveguides 292
9.6 Discontinuities in circular waveguides 297

10 Further applications of the W-H technique 301

10.1 The step problem 301
10.1.1 Deduction of the transverse modified W-H equations (E-polarization case) 301
10.1.2 Solution of the equations 303
10.2 The strip problem 303
10.2.1 Some longitudinally modified W-H geometries 304
10.3 The hole problem 304
10.4 The wall problem 305
10.5 The semi-infinite duct with a flange 307

10.6 Presence of dielectrics 308
10.7 A problem involving a dielectric slab 310
10.8 Some problems involving dielectric slabs 313
10.8.1 Semi-infinite dielectric guides 314
10.8.2 The junction of two semi-infinite dielectric slab guides 314
10.8.3 Some problems solved in the literature 314
10.9 Some problems involving periodic structures 315
10.9.1 Diffraction by an infinite array of equally spaced half-planes immersed in free space 315
10.9.2 Other problems solved in the literature 317
10.10 Diffraction by infinite strips 318
10.10.1 Solution of the key problem 319
10.10.2 Boundary conditions 321
10.10.3 Solution of the W-H equation 321
10.11 Presence of an inductive iris in rectangular waveguides 323
10.12 Presence of a capacitive iris in rectangular waveguides 324
10.13 Problems involving semi-infinite periodic structures 324
10.14 Problems involving impedance surfaces 325
10.15 Some problems involving cones 326
10.16 Diffraction by a PEC wedge by an incident plane wave at skew incidence 330
10.17 Diffraction by a right PEC wedge immersed in a stratified medium 334
10.18 Diffraction by a right isorefractive wedge 337
10.18.1 Solution of the W-H equations 342
10.18.2 Matrix factorization of $g_e(\alpha)$ 345
10.18.3 Near field behavior 347
10.19 Diffraction by an arbitrary dielectric wedge 349

References 351
Index 361