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1 The inverted pendulum: history and survey of open and current problems in control theory and robotics

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